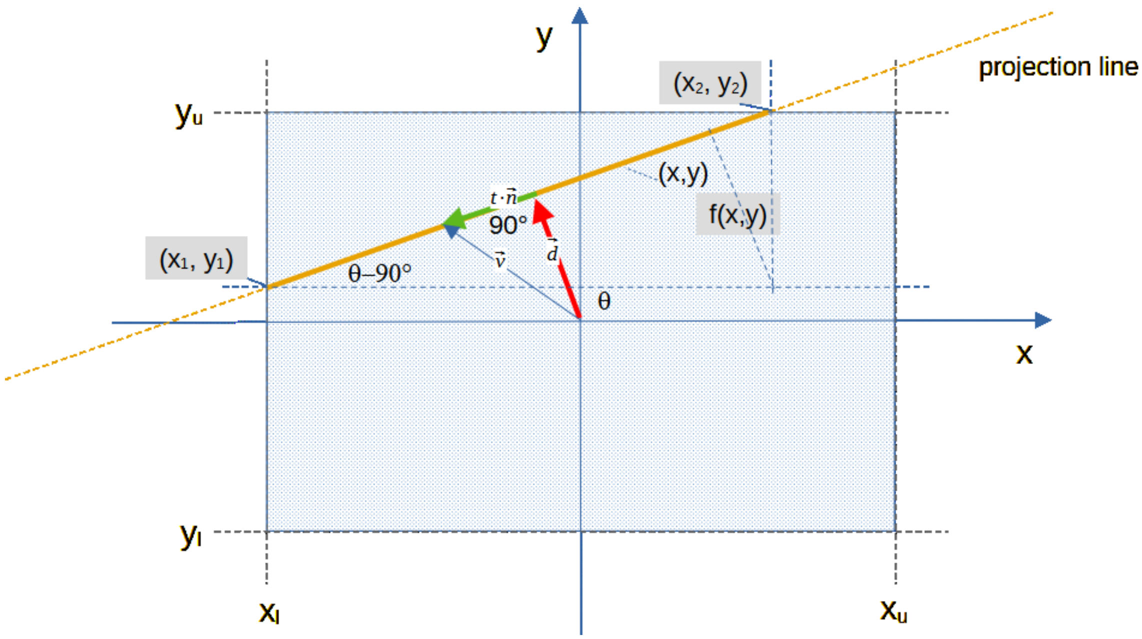
Computing the Projection(s) of an Image

This notebook reviews some methods to compute the projections of a 2D image. The concept is explained by the figure below.



The figure is bounded by a rectangular region which is bounded by x_l, y_u, x_u and y_l (left, top, right, bottom).

The line (projection line) intersects the figure at points x_1, y_1 and x_2, y_2. Integrating over the figure along this line yields the projection for this particular projection line.

While there are quite a few ways to define the projection line a frequently use approach is to express the line by a vector equation like this:

$$\vec{v} = \begin{pmatrix} x \\ y \end{pmatrix} = \underbrace{d \cdot \begin{pmatrix} cos\left(\theta\right) \\ sin\left(\theta\right) \end{pmatrix}}_{\vec{d}} + \underbrace{t \cdot \begin{pmatrix} -sin\left(\theta\right) \\ cos\left(\theta\right) \end{pmatrix}}_{\vec{n}}$$

With this formulation of the projection line the projection $P\left(d, \theta\right)$ is computed via the integral

$$P\left(d, \theta\right) = \int_{t_1}^{t_2} f\left(d \cdot cos\left(\theta\right) - t \cdot sin\left(\theta\right), d \cdot sin\left(\theta\right) + t \cdot cos\left(\theta\right)\right) \cdot dt$$

Note

With projections $P\left(d, \theta\right)$ for many values pairs d, θ (also referred to as Radon transform) it is possible to reconstruct the image.

For a point x_p, y_p the values of d and t are determined for a given angle θ.

$$d = x_p \cdot cos\left(\theta\right) + y_p \cdot sin\left(\theta\right)$$

$$t = -x_p \cdot sin\left(\theta\right) + y_p \cdot cos\left(\theta\right)$$

The last equation is used to determine the integration limits t_1 and t_2.

$$t_1 = -x_1 \cdot sin\left(\theta\right) + y_1 \cdot cos\left(\theta\right)$$

$$t_2 = -x_2 \cdot sin\left(\theta\right) + y_2 \cdot cos\left(\theta\right)$$

Computing Intersections

To compute the projection along a line defined by parameters d and θ we need to compute the intersection points $x_1,\ y_1$ and $x_2,\ y_2$.

If the line intersects with the rectangle, an intersection may occur for these cases:

case#1 (left)

intersection occurs on the *left side* of the rectangle for $x = x_l$ and a specific value y in the range $y_l \leq y \leq y_u$.

$$x_l = d \cdot cos\left(\theta\right) - t_{c1} \cdot sin\left(\theta\right)$$

with t_{c1} :

$$t_{c1} = \frac{d \cdot cos\left(\theta\right) - x_l}{sin\left(\theta\right)}$$

$$y = d \cdot sin\left(\theta\right) + t_{c1} \cdot cos\left(\theta\right)$$

If y is in the interval $y_l \leq y \leq y_u$ then we have an intersection.

case#2 (right)

intersection occurs on the *right side* of the rectangle for $x = x_u$ and a specific value y in the range $y_l \leq y \leq y_u$.

$$x_u = d \cdot cos\left(\theta\right) - t_{c2} \cdot sin\left(\theta\right)$$

with t_{c2} :

$$t_{c2} = \frac{d \cdot cos\left(\theta\right) - x_u}{sin\left(\theta\right)}$$

$$y = d \cdot sin\left(\theta\right) + t_{c2} \cdot cos\left(\theta\right)$$

If y is in the interval $y_l \leq y \leq y_u$ then we have an intersection.

case#3 (top)

intersection occurs on the *top side* of the rectangle for $y = y_u$ and a specific value x in the range $x_l \leq x \leq x_u$.

$$y_u = d \cdot sin\left(\theta\right) + t_{c3} \cdot cos\left(\theta\right)$$

with t_{c3} :

$$t_{c3} = \frac{y_u - d \cdot sin\left(\theta\right)}{cos\left(\theta\right)}$$

$$x = d \cdot cos\left(\theta\right) - t_{c3} \cdot sin\left(\theta\right)$$

If x is in the interval $x_l \leq x \leq x_u$ we have an intersection.

case#4 (bottom)

intersection occurs on the *bottom side* of the rectangle for $y = y_l$ and a specific value x in the range $x_l \leq x \leq x_u$.

$$y_l = d \cdot sin\left(\theta\right) + t_{c4} \cdot cos\left(\theta\right)$$

with t_{c4} :

$$t_{c4} = \frac{y_l - d \cdot sin\left(\theta\right)}{cos\left(\theta\right)}$$

$$x = d \cdot cos\left(\theta\right) - t_{c4} \cdot sin\left(\theta\right)$$

If x is in the interval $x_l \leq x \leq x_u$ we have an intersection.

Special cases are $\theta = 0$ and $\theta = \frac{\pi}{2}$.

case#5: $\theta = 0$ (vertical projection line)

For x in $x_l \leq x \leq x_u$ intersections occur at points x, y_l and x, y_u.

case#6: $\theta = \frac{\pi}{2}$ (horizontal projection line)

For y in $y_l \leq y \leq y_u$ intersections occur at points x_l, y and x_u, y.

Example

The code to compute intersections is in Python file `intersections.py`.

For a pair of values d, θ the procedure checks for the conditions formulated for cases #1 to #6. Either no intersection can be found of two intersections can be found.

To demonstrate the application of function `intersections.py` a rectangle is defined with corners at $(x_l = -5, y_l = -6)$, $(x_l = -5, y_u = 7)$, $(x_u = 4, y_l = -6)$ and $(x_u = 4, y_u = 7)$. (The image is represented in a physical coordinate system (x,y) as opposed to image coordinates which are frequently used in the representation of discrete / digital images (eg. captured from a digital camera).

The rectangle is displayed along with the intersections and the projection line which interconnects the intersections.

In [23]:
```
#%matplotlib inline
import sys, os
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.patches as pat
import math
sys.path.append(os.path.join(os.getcwd(), 'modules'))
import intersections as isec
import cv2
```

In [24]:
```
# the rectangular region
x_l = -5
x_u = 4.0
y_l = -6.0
y_u = 7.0

# line parameters
d = 4.5
theta_deg = 171.0
dx = d * math.cos(math.radians(theta_deg))
dy = d * math.sin(math.radians(theta_deg))
```

In [25]:
```
# compute intersections if there are any ...
pCount, iPoints = isec.intersections(d, theta_deg, x_l, x_u, y_l, y_u)
print(f"pCount  : {pCount}")
print(f"iPoints : {iPoints}")
```

```
pCount  : 2
iPoints : [[-5, -2.802718076626738], [-3.4474019837742578, 7.0]]
```

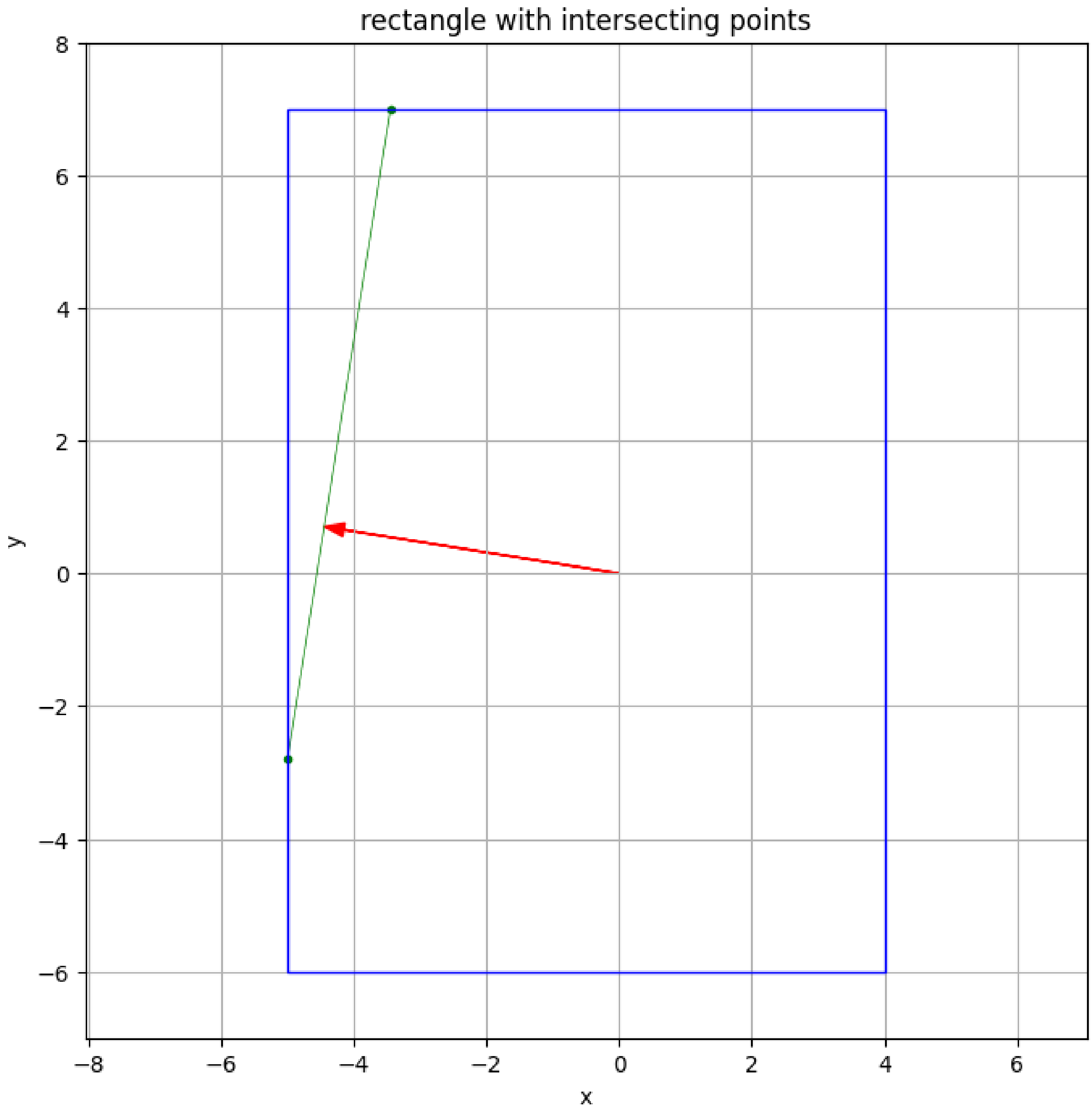
In [26]:
```
if pCount == 2:
    # tuple (x1, x2)
    xvec = [iPoints[0][0], iPoints[1][0]]
    # tuple (y1, y2)
    yvec = [iPoints[0][1], iPoints[1][1]]
    # integration limits (t1, t2)
    t1 = -xvec[0] * math.sin(math.radians(theta_deg)) + yvec[0] * math.cos(math.rad
    t2 = -xvec[1] * math.sin(math.radians(theta_deg)) + yvec[1] * math.cos(math.rad
    delta_x = abs(xvec[0] - xvec[1])
    delta_y = abs(yvec[0] - yvec[1])
    len_projection = math.sqrt(delta_x**2 + delta_y**2)
    # len_projection should be (t2 - t1)
    print(f"t1: {t1}, t2: {t2}, t2 - t1: {t2-t1} ,  len_projection: {len_projection
```

```
t1: 3.5503842914606136, t2: -6.374525899055607, t2 - t1: -9.92491019051622 ,  len_pr
ojection: 9.92491019051622
```

Display Intersection (if available)

1. The figure below shows the rectangle bounded by a blue boundary.
2. Vector $\vec{d}$ is shown as a red arrow.
3. The part of the intersecting line which is within the rectangle is displayed in green.
4. Green dots indicate the intersection points x_1, y_1 and x_2, y_2

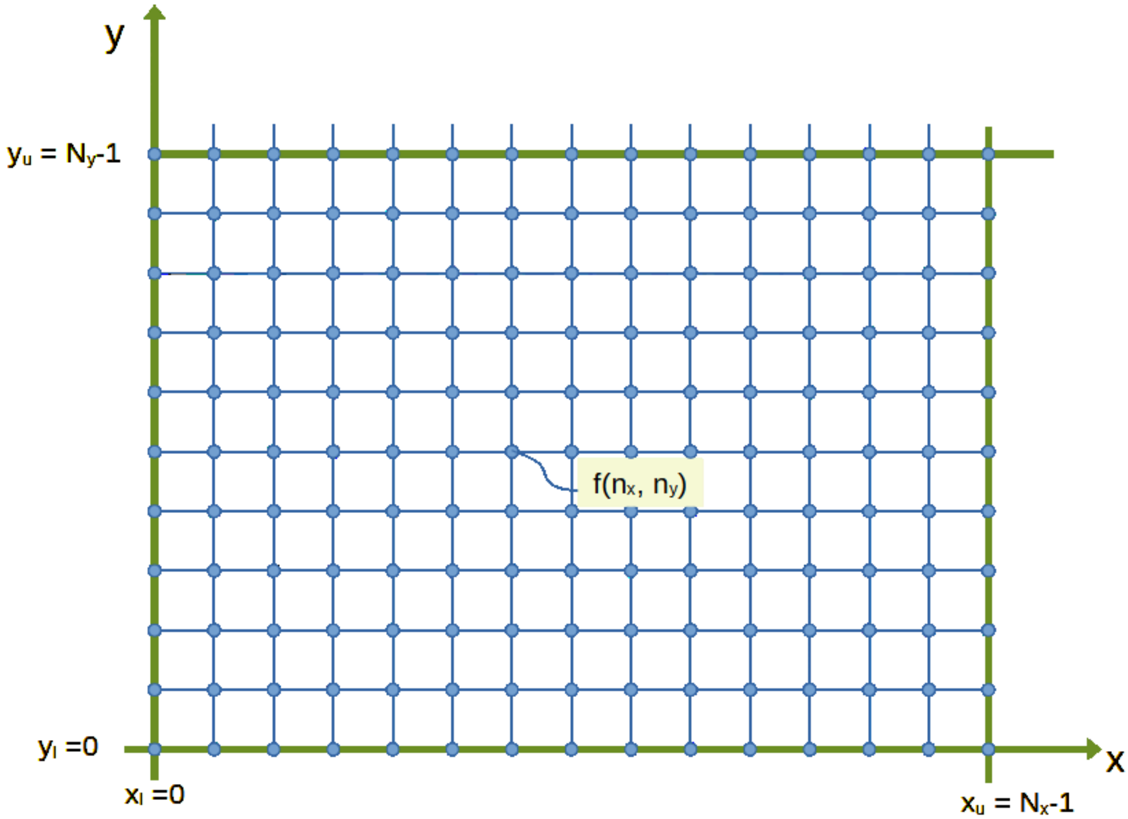
In [27]:
```
fig1 = plt.figure(1, figsize=[8, 8])
ax_f1 = fig1.add_subplot(1, 1, 1)
ax = plt.gca()
ax_f1.add_patch(pat.Rectangle( (x_l, y_l), width=(x_u - x_l), height=(y_u - y_l), e
# ax_f1.legend()

# plot arrow
ax_f1.arrow(0, 0, dx, dy, color='r', length_includes_head=True, head_width=0.2)

# plot intersecting line
if pCount == 2:
    ax_f1.plot(xvec, yvec, color='g', linewidth=0.5, marker='o', markersize=3)

ax_f1.axis('equal')
ax_f1.grid(True)
ax_f1.set_xlabel('x')
ax_f1.set_ylabel('y')
ax_f1.set_title('rectangle with intersecting points');
ax_f1.set_xlim(x_l - 1, x_u + 1)
ax_f1.set_ylim(y_l - 1, y_u + 1);
```



Computing the Projection for a discrete Image

The definition of a projection $P\left(d,\theta\right)$ along a line has been defined for a continous function / image $f(x,y)$.

Now we discuss the case where the image $f(x,y)$ is defined for discrete points on a rectangular grid with N_y rows and N_x columns. Thus the image has a total of $N_y \cdot N_x$ points.



Here we are using **physical** coordinates for the discrete image points $f(n_x, n_y)$. If images are loaded from a file the image points are represented in a **image** coordinate system. Then an image is represented in matrix notation (row, col)-order with the upper left corner of the image at $f(0, 0)$.

Definition of a Projection

Being able to compute the intersection points we can now compute the projection of an image along a straight line defined by parameters d, θ. A discretized image will be assumed. With N_y pixel in y-direction and N_x pixels in x direction the image boundaries are as follows:

1. left border: $x_l = 0$ and $0 \leq y \leq N_y - 1$
2. top / upper border: $y_u = N_y - 1$ and $0 \leq x \leq N_x - 1$
3. right border: $x_u = N_x - 1$ and $0 \leq y \leq N_y - 1$
4. bottom border: $y_l = 0$ and $0 \leq x \leq N_x - 1$

The procedure to compute a projection is outlined here:

1. for parameters d, θ, compute whether the line intersects the figure. If an intersection occurs return the number of intersections (must be 2 of course) and the intersection points x_1, y_1 and x_2, y_2. If there is no intersection return the number of intersections as

0 and the intersection points as None values. Function `intersections.py` computes the intersection points.

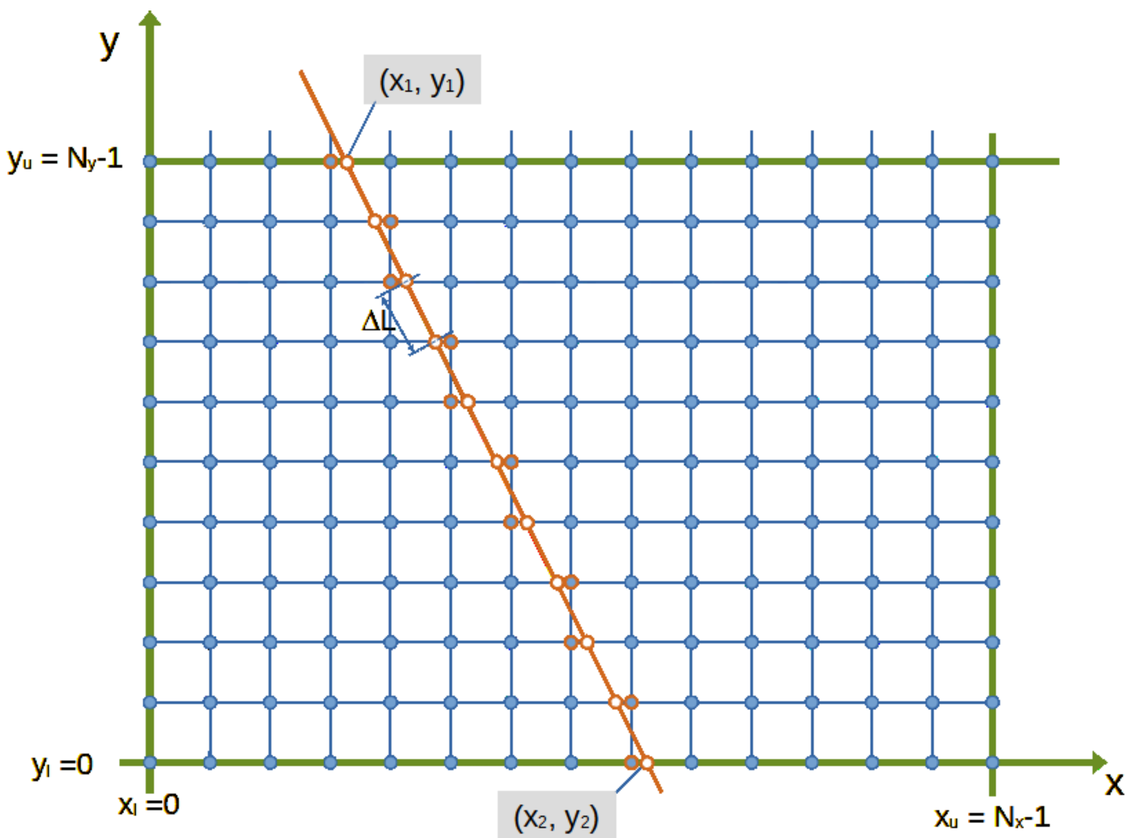
2. If intersection points x_1, y_1 and x_2, y_2 have been found, collect the row / column indices along the *projection line* which interconnects the intersection points. Function `projectionIndices` computes the row / column indices of a projection line.

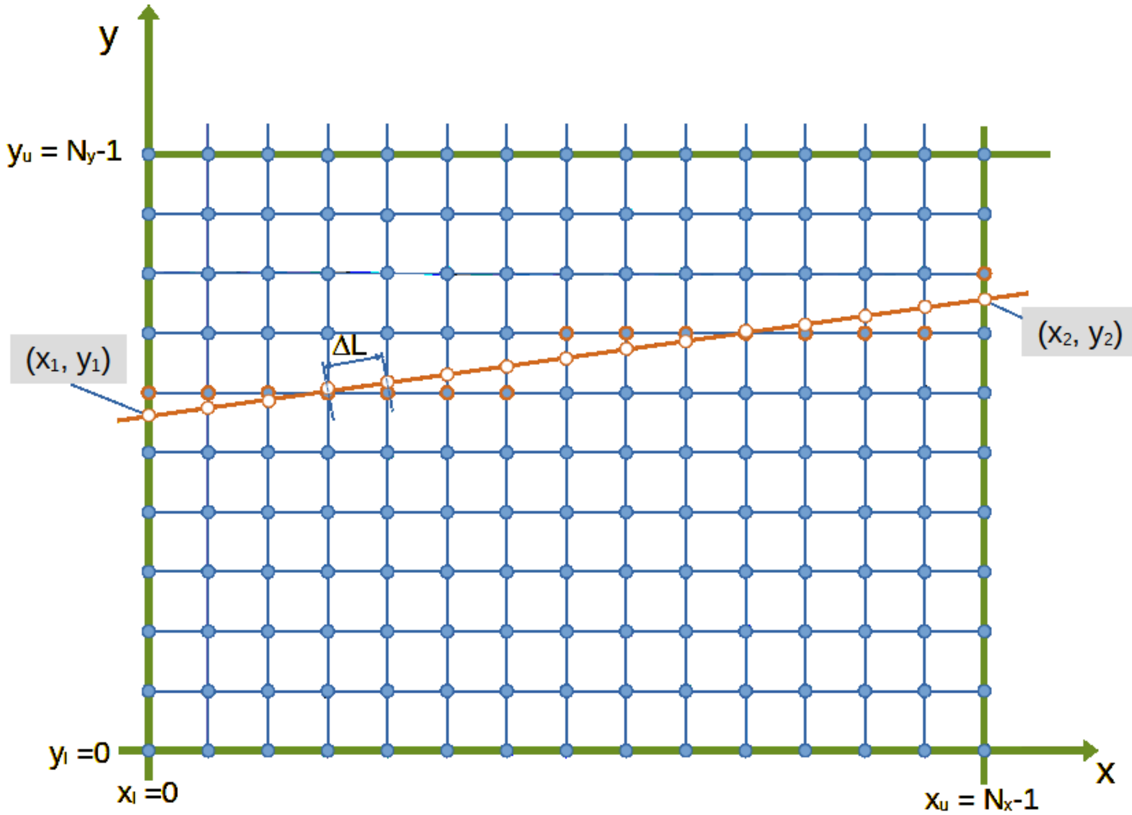
3. Summing up all pixel values of the image along the projection line yields the value of the projection. The row / column indices computed from function `projectionIndices` are used to select the pixels pertaining to the projection line. Function `projectionSingleLine` computes the value of the projection using functions `intersections.py` and `projectionIndices`

Two figures illustrate the procedure of collecting the appropriate row / column indices along a projection line.

The first figure shows a case, where y coordinate changes by larger amount than the x coordinate when moving along the projection line from pixel to pixel. A second figure shows the opposite situation. Here the x coordinate changes by a larger amount than y when moving from pixel to pixel.

No interpolation of pixel values is used to keep computations simple.





Application to an Image

The use of functions `intersections(...)` and `projectionIndices(...)` shall be demonstrated using a simple image. The image is computed in Jupyter notebook `test_images_rd1.ipynb` and stored in file.

1. Load image from file
2. compute intersections for a set of d and θ.
3. plot intersections using function `intersections(...)`
4. plot the line connection the intersection points
5. plot the projection line computed from function `projectionIndices(...)`

Loading file

1. Determine the shape of image data `Nx` number of columns and `Ny` the number of rows
2. Define the rectangular region which encloses the image

   A. the rectangular boudaries of the image are determines by 4 points:

a. lower left corner: (x_l, y_l)

b. upper left corner (x_l, y_u)

c. lower right corner: (x_u, y_l)

d. upper right corner: (x_u, y_u)

Note

At this point *no* image coordinates are used. (image coordinates have the upper left corner at (0,0))

In [28]:
```
# load file
imgFile = "images/testImgRect1.npy"
img = np.load(imgFile)
Nx = img.shape[1]
Ny = img.shape[0]
print(f"size of image: {img.size} ; shape of image: {img.shape}")
```

```
size of image: 1800000 ; shape of image: (1000, 1800)
```

In [29]:
```
# Define boundaries of the rectangular region
x_l = 0  # left x
x_u = Nx - 1 # right x
y_l = 0 # bottom y
y_u = Ny - 1 # top y
```

Compute a projection line

Recipe

1. For fixed parameters d and θ the intersecting points (if there are any) of the the projection line are determined
2. The projection line is defined by an array of x-Indices `indexX` an an array of y-Indices `indexY`

In [30]:
```
# line parameters
d = 1000
phi_deg = 40.0

# compute intersections
pCount, iPoints = isec.intersections(d, phi_deg, x_l, x_u, y_l, y_u)

if len(iPoints) == 2:
    # intersection
    x1 = iPoints[0][0]
    y1 = iPoints[0][1]
    x2 = iPoints[1][0]
    y2 = iPoints[1][1]
    indexX, indexY = isec.projectionIndices(iPoints, Nx, Ny)
```

```
    dx = x2 - x1
    dy = y2 - y1
    if dx != 0:
        slope = dy/dx
    else:
        slope = math.nan

    print(f"slope : {slope}")
    print(f"pCount  : {pCount}")
    print(f"iPoints : {iPoints}")
    print(f"dx : {dx} ; dy : {dy}")
```

```
slope : -1.1917535925942102
pCount  : 2
iPoints : [[467.1467577861758, 999], [1305.4072893322784, 0]]
dx : 838.2605315461026 ; dy : -999
```

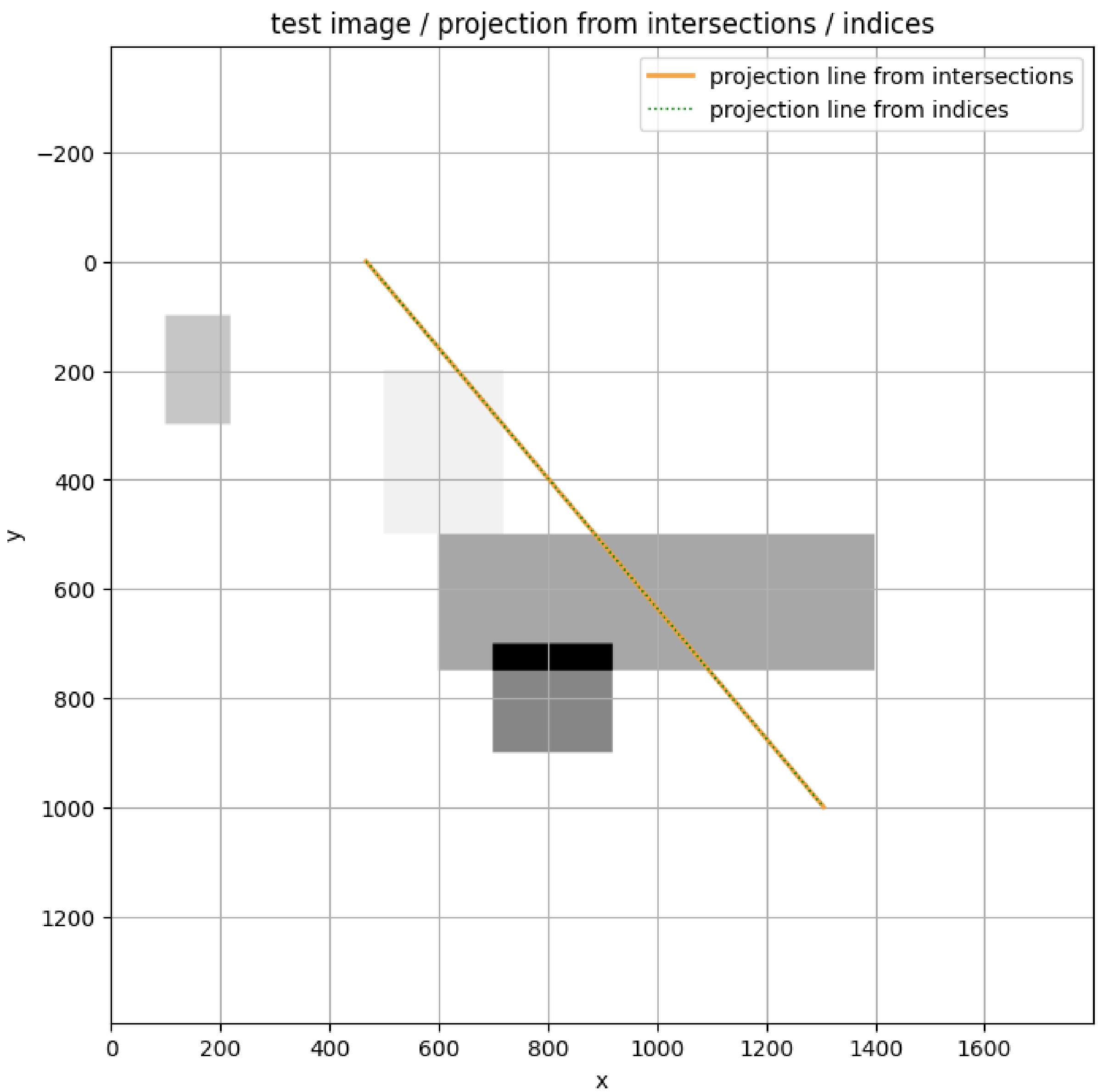
Applying corrections for image coordinates

The image is displayed in image coordinates with x=0 y=0 being the upper left corner of the image. However the intersection points and indices of the projection line have been computed in physical x/y coordinates. Before plotting the intersection points and the projection line the physical coordinates must be transformed to image coordinates.

The transformation only affects the physicaly coordinate. It is is flipped *up / down* using `Ny -1 - (physical_Y_coordinates)`

The figure below shows an excellent match of both variants of projection lines.

In [31]:
```
fig2 = plt.figure(2, figsize=[8, 8])
ax_f2 = fig2.add_subplot(1, 1, 1)

# plot of image
ax_f2.imshow(img, cmap='Greys' )
ax_f2.axis('equal')
# plot projection line from intersections, the y coordinates must be transformed to
ax_f2.plot([x1, x2], [Ny - 1 - y1, Ny - 1 - y2], linewidth=2, color='#f5a142', labe
# plot projection line from indices indexX and indexY; again y coordinates must be
ax_f2.plot(indexX, Ny -1 - indexY, linewidth=1, color='g', linestyle=':', label='pr
ax_f2.grid(True)
ax_f2.set_xlabel('x')
ax_f2.set_ylabel('y')
ax_f2.legend()
ax_f2.set_title('test image / projection from intersections / indices');
```



Computing Projections

1. define a fixed angle θ for which projections shall be computed
2. define an array of d-values
3. for each d-value compute a projection line and the accumulated value along that line
4. display the image and the projection lines
5. display the projection as a function of d-values for a constant angle θ.

In [32]:
```
# the rectangular region
x_l = 0
x_u = Nx - 1
```

```python
y_l = 0
y_u = Ny - 1

# line parameters
d_min = -2000
d_max = 2000
Nd = 200
# array of d-values
dVec = np.linspace(d_min, d_max, Nd)
# fixed angle
theta_deg = 30.0

# compute projection for elements of dVec and fixed angle theta_deg
projections = isec.projectionMultiLine(dVec, theta_deg, img, x_l, x_u, y_l, y_u, Nx
```

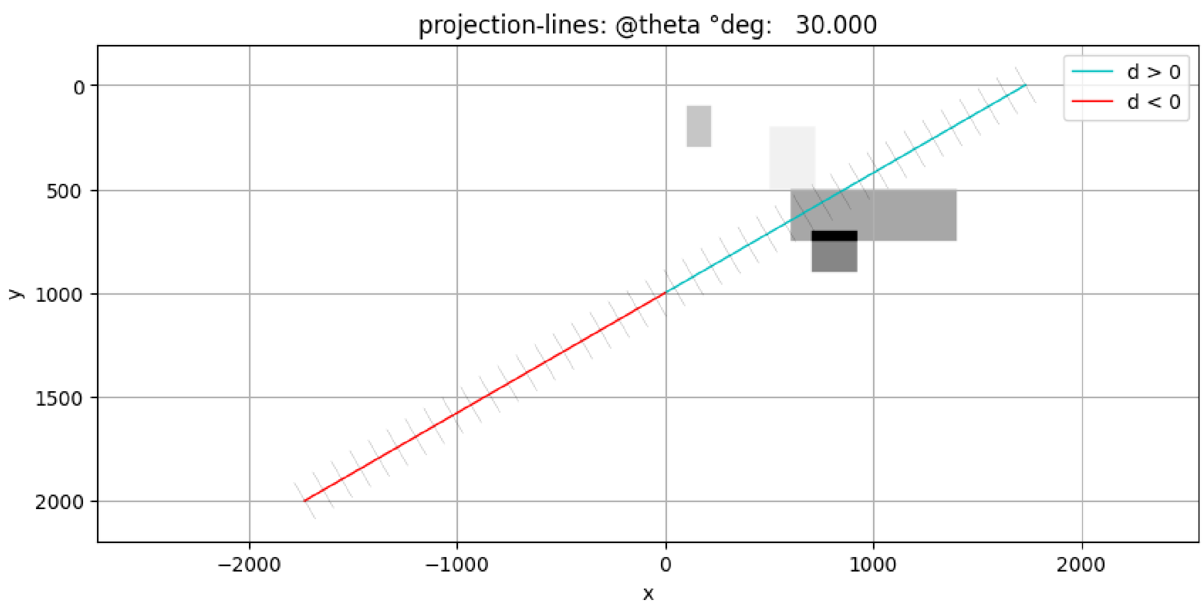
In [33]:

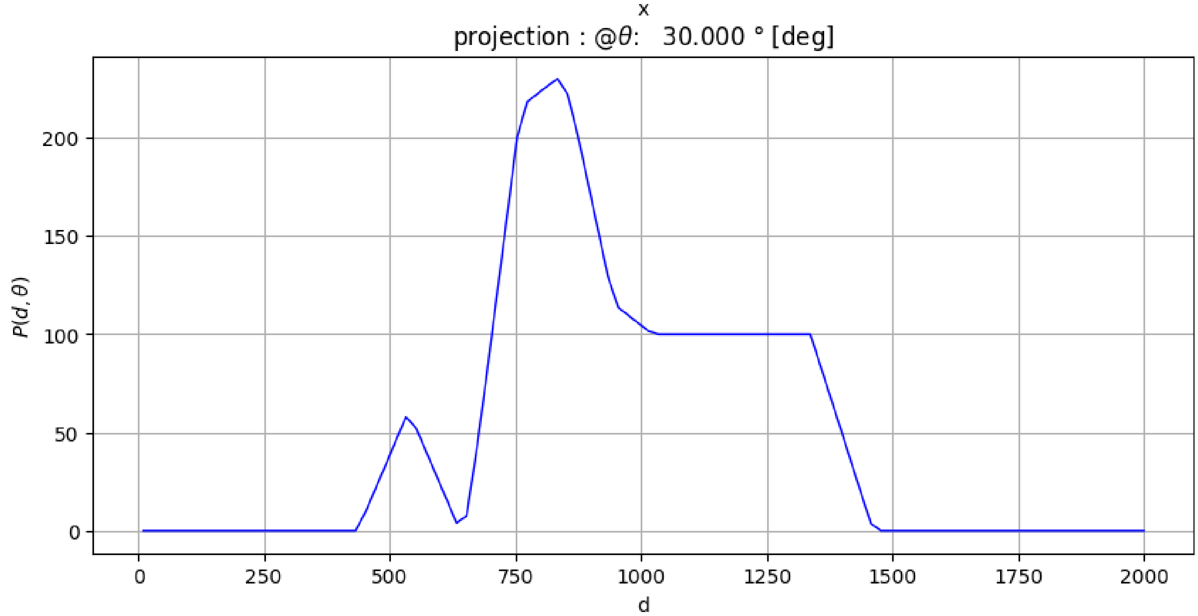
```python
fig3 = plt.figure(3, figsize=[10, 10])
ax_f31 = fig3.add_subplot(2, 1, 1)
ax_f31.imshow(img, cmap='Greys' )

ax_f31.axis('equal')

# unit d-vector (dx, dy)
dx = math.cos(math.radians(theta_deg))
dy = math.sin(math.radians(theta_deg))
# unit n-vector (nx, ny)
nx = -math.sin(math.radians(theta_deg))
ny = math.cos(math.radians(theta_deg))

# projection lines
d_xmax = d_max * dx
d_y_xmax = d_max * dy
d_xmin = d_min * dx
d_y_xmin = d_min * dy

if d_max > 0:
    ax_f31.plot([0, d_xmax], [Ny - 1, Ny - 1 - d_y_xmax], linewidth=1, color='c', l
else:
    ax_f31.plot([0, d_xmax], [Ny - 1, Ny - 1 - d_y_xmax], linewidth=1, color='r', l

if d_min > 0:
    ax_f31.plot([0, d_xmin], [Ny - 1, Ny - 1 - d_y_xmin], linewidth=1, color='c', l
else:
    ax_f31.plot([0, d_xmin], [Ny - 1, Ny - 1 - d_y_xmin], linewidth=1, color='r', l

t = 100
nPLines = 40

for d in np.linspace(d_min, d_max, num=nPLines):
    # point (x1, y1)  on projection line
    x1 = d*dx - t*nx
    y1 = d*dy - t*ny
    # point (x2, y1)  on projection line
    x2 = d*dx + t*nx
    y2 = d*dy + t*ny
    ax_f31.plot([x1, x2], [Ny - 1 - y1, Ny - 1 - y2], linewidth=0.1, color='k')
```

```
ax_f31.grid(True)
ax_f31.set_xlabel('x')
ax_f31.set_ylabel('y')
ax_f31.legend()
ax_f31.set_title(f"projection-lines: @theta °deg: {theta_deg:8.3f}")

# plot projection as a subplot
ax_f32 = fig3.add_subplot(2, 1, 2)
ax_f32.plot(dVec, projections, linewidth=1, color='b', linestyle='-')
ax_f32.grid(True)
ax_f32.set_xlabel('d')
ax_f32.set_ylabel('$P(d, \\theta)$')
ax_f32.set_title(f"projection : @$\\theta$: {theta_deg:8.3f} ° [deg]")
```

Out[33]:

```
Text(0.5, 1.0, 'projection : @$\\theta$:    30.000 ° [deg]')
```





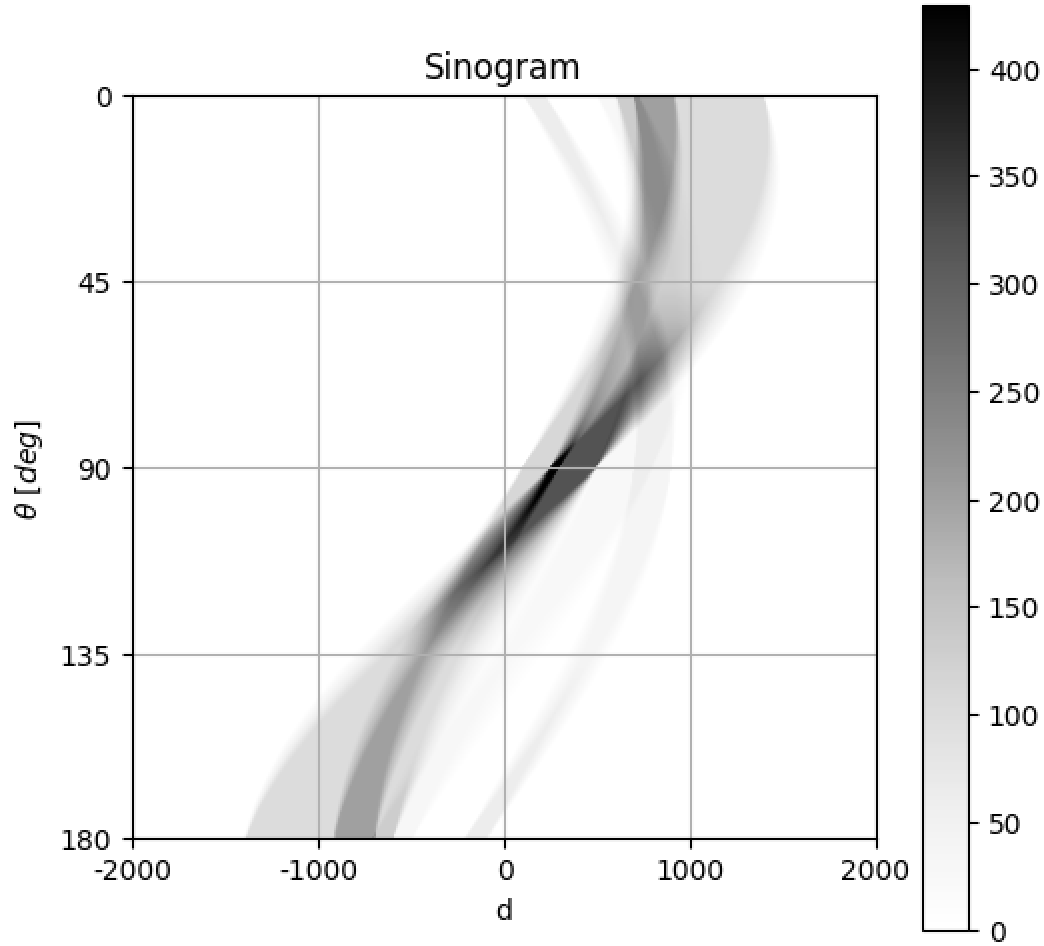
Computing a Sinogram

A sinogram is the display of all projection values versus a set of N_d values of d and N_{theta} values of angle θ.

From the image used previously the sinogram is computed and displayed as an image.

In [34]:
```
d_min = -2000
d_max = 2000
Nd = 400
dVec = np.linspace(d_min, d_max, Nd)

Ntheta = 400
thetaVec_deg = np.linspace(0, 179, Ntheta)

# initalise matrix
sinogram = np.zeros((Ntheta, Nd), dtype=np.float64)

for nc, theta in enumerate(thetaVec_deg):
    # compute projection
    sinogram[nc, :] = isec.projectionMultiLine(dVec, theta, img, x_l, x_u, y_l, y_u
```

In [35]:
```
fig4 = plt.figure(4, figsize=[6, 6])
ax_f41 = fig4.add_subplot(1, 1, 1)
# a = ax_f41.imshow(sinogram, cmap='hot' )
a = ax_f41.imshow(sinogram, cmap='Greys' )

ax_f41.grid(True)
ax_f41.set_xlabel('d')
ax_f41.set_ylabel('$\\theta \ [deg]$ ')
ax_f41.set_title(f"Sinogram")

yticks = [0, 100, 200, 300, 399]
ytickLabels = ['0', '45', '90', '135', '180']
ax_f41.set_yticks(yticks, ytickLabels)

xticks = [0, 100, 200, 300, 400]
xtickLabels = ['-2000', '-1000', '0', '1000', '2000'];
ax_f41.set_xticks(xticks, xtickLabels)

fig4.colorbar(a, ax=ax_f41, location='right');
```



Why is it called a Sinogram ?

Consider all projection lines defined by d, θ which pass through a point at x_p, y_p. Assume that the value $f(x_p, y_p)$ at this point has a value of v and all other image points have *zero* value (point image). Then all projections have a value of v.

From the equation

$$\begin{pmatrix} x_p \\ y_p \end{pmatrix} = d \cdot \begin{pmatrix} cos\,(\theta) \\ sin\,(\theta) \end{pmatrix} + t \cdot \begin{pmatrix} -sin\,(\theta) \\ cos\,(\theta) \end{pmatrix}$$

the values of d and t are determined for a given angle θ.

$$d = x_p \cdot cos\,(\theta) + y_p \cdot sin\,(\theta)$$

$$t = -x_p \cdot sin\,(\theta) + y_p \cdot cos\,(\theta)$$

For our purpose only the equation for d is interesting. To gain more insight the equation is rewritten:

$$d = \sqrt{x_p^2 + y_p^2} \cdot \left(\frac{x_p}{\sqrt{x_p^2 + y_p^2}} \cdot cos\left(\theta\right) + \frac{y_p}{\sqrt{x_p^2 + y_p^2}} \cdot sin\left(\theta\right) \right)$$

Using definitions

$$sin\left(\phi\right) = \frac{x_p}{\sqrt{x_p^2 + y_p^2}}$$

$$cos\left(\phi\right) = \frac{y_p}{\sqrt{x_p^2 + y_p^2}}$$

and

$$tan\left(\phi\right) = \frac{sin\left(\phi\right)}{cos\left(\phi\right)} = \frac{x_p}{y_p}$$

the equation for d can now be written more compactly as:

$$d = \sqrt{x_p^2 + y_p^2} \cdot \left(sin\left(\phi\right) \cdot cos\left(\theta\right) + cos\left(\phi\right) \cdot sin\left(\theta\right)\right)$$

and finally as:

$$d = \sqrt{x_p^2 + y_p^2} \cdot sin\left(\theta + \phi\right)$$

Summary

A single point x_p, y_p of a figure $f(x,\ y$ is transformed into a *sinogram* which is *sinusoidal* dependent on angle θ. The phase constant phase ϕ depends on point coordinates with $\phi = arctan\left(\frac{x_p}{y_p}\right)$

Demonstration of a Sinogram of an Image with 3 Points

The sinogram of an image with only 3 non-zero pixels is computed with function `projectionMultiLine` . The sinogram is displayed. It is composed of 3 sinusoids.

Additionally the sinogram of one of the 3 points is computed *analytically*. Apparently the analytical result matches the computation with `projectionMultiLine` .

In [36]:

```
# the rectangular region
Ny2 = 200
Nx2 = 200

# physical dimension of region
x_l2 = 0
```

```
x_u2 = Nx2 - 1
y_l2 = 0
y_u2 = Ny2 - 1

# parameters for d-vector and angle vector
d_min2 = -300
d_max2 = 300
Nd2 = 600
dVec2 = np.linspace(d_min2, d_max2, Nd2)
Ntheta2 = 400
thetaVec2_deg = np.linspace(0, 179, Ntheta2)

# image: 3 points
img2 = np.zeros((Ny2, Nx2), dtype=np.float64)
# colum , row of point1 (p1)
nc_p1 = 50
nr_p1 = 50
# colum , row of point2 (p2)
nc_p2 = 150
nr_p2 = 130
# colum , row of point3 (p3)
nc_p3 = 100
nr_p3 = 180

img2[nr_p1, nc_p1] = 1
img2[nr_p2, nc_p2] = 1
img2[nr_p3, nc_p3] = 1

# initalise matrix of sinogram
sinogram2 = np.zeros((Ntheta2, Nd2), dtype=np.float64)

for ncol, theta in enumerate(thetaVec2_deg):
    # compute projection
    sinogram2[ncol, :] = isec.projectionMultiLine(dVec2, theta, img2, x_l2, x_u2, y
```

In [37]:

```
# compute sinogram for single point1
# compute physical coordinates
x_p1 = nc_p1
y_p1 = Ny2 - 1 - nr_p1

R = math.sqrt(x_p1**2 + y_p1**2)
A = x_p1/R
B = y_p1/R

# corresponding d values
da_p1 = R * (A * np.cos(np.pi * thetaVec2_deg/180) + B * np.sin(np.pi * thetaVec2_d
```

In [38]:

```
fig_width = 10
fig_height = 10
fig5 = plt.figure(5, figsize=[fig_width, fig_height])

ax_f51 = fig5.add_subplot(2, 1, 1)
ax_f51.imshow(img2, cmap='binary')
ax_f51.set_title(f"image with 3 points")
```

```
ax_f52 = fig5.add_subplot(2, 1, 2)
a = ax_f52.imshow(sinogram2, cmap='binary')

# superimpose analytical computed sinogram of point1 with sinogram2
# plot only every 10'th item to avoid masking sinogram2 (3 points)
# -> excellent match ...
d_offset = 300
scale = Ntheta2/180
ax_f52.plot(da_p1[::10] + 300, scale * thetaVec2_deg[::10], linestyle='none', marke

ax_f52.grid(True)
ax_f52.set_xlabel('d')
ax_f52.set_ylabel('$\\theta \ [deg]$ ')
ax_f52.set_title(f"Sinogram of 3 points & analytical results")
ax_f52.legend()

yticks2 = [0, 100, 200, 300, 399]
ytickLabels2 = ['0', '45', '90', '135', '180']
ax_f52.set_yticks(yticks2, ytickLabels2)

xticks2 = [0, 150, 300, 450, 600]
xtickLabels2 = ['-300', '-150', '0', '150', '300'];
ax_f52.set_xticks(xticks2, xtickLabels2);
```

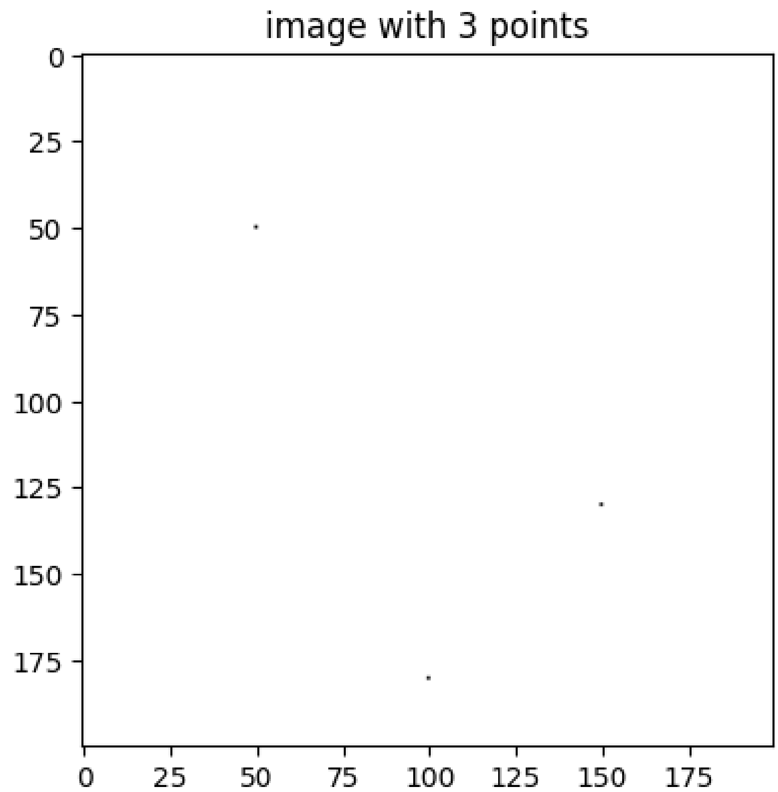
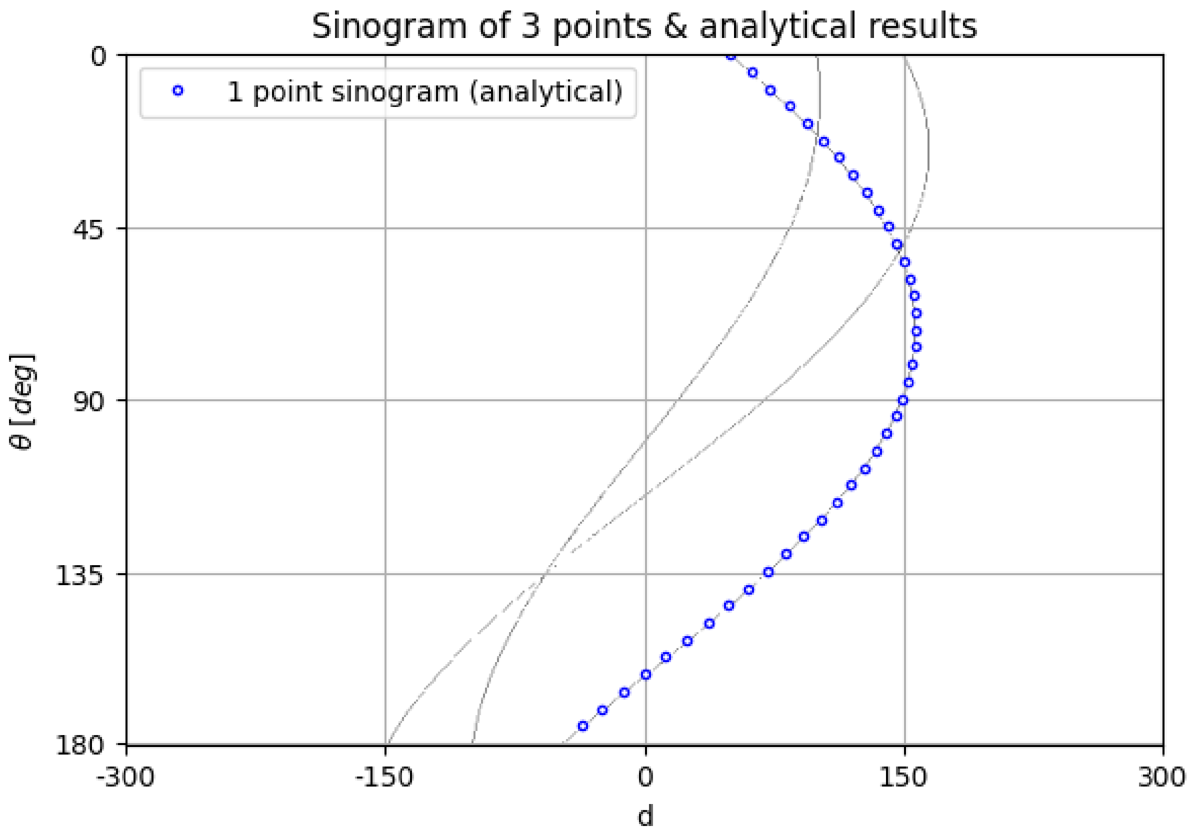


image with 3 points
0
25
50
75
100
125
150
175
0
25
50
75
100
125
150
175


Sinogram of 3 points & analytical results
1 point sinogram (analytical)
0
45
90
135
180
θ [deg]
-300
-150
0
150
300
d

Other Examples

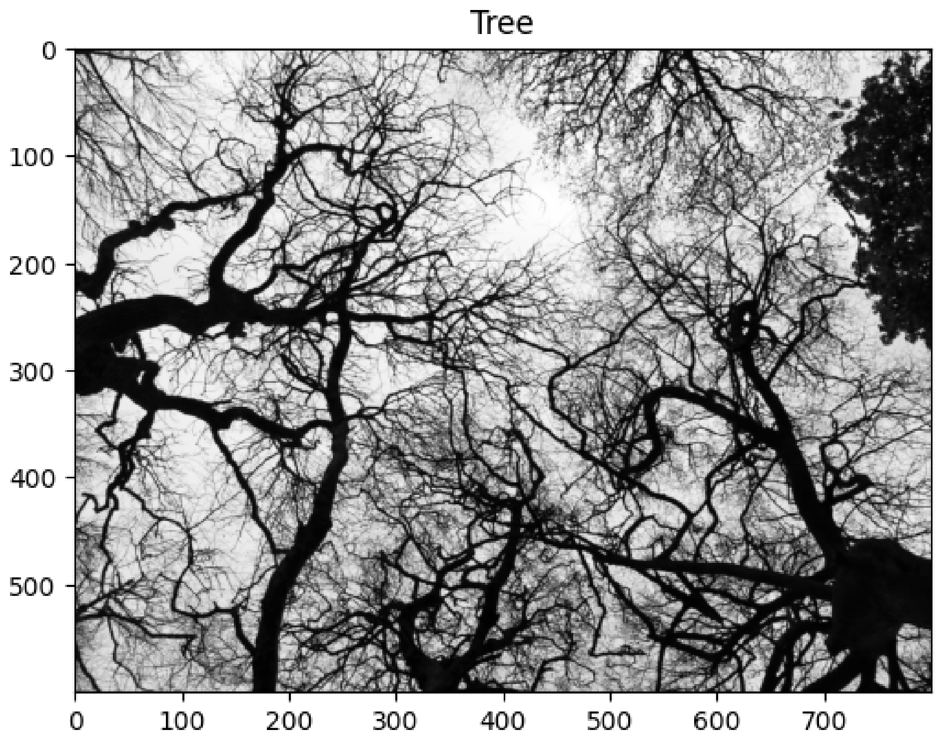
In these examples the sinograms are computed from grayscale images which exhibit some structure (eg.: lines).

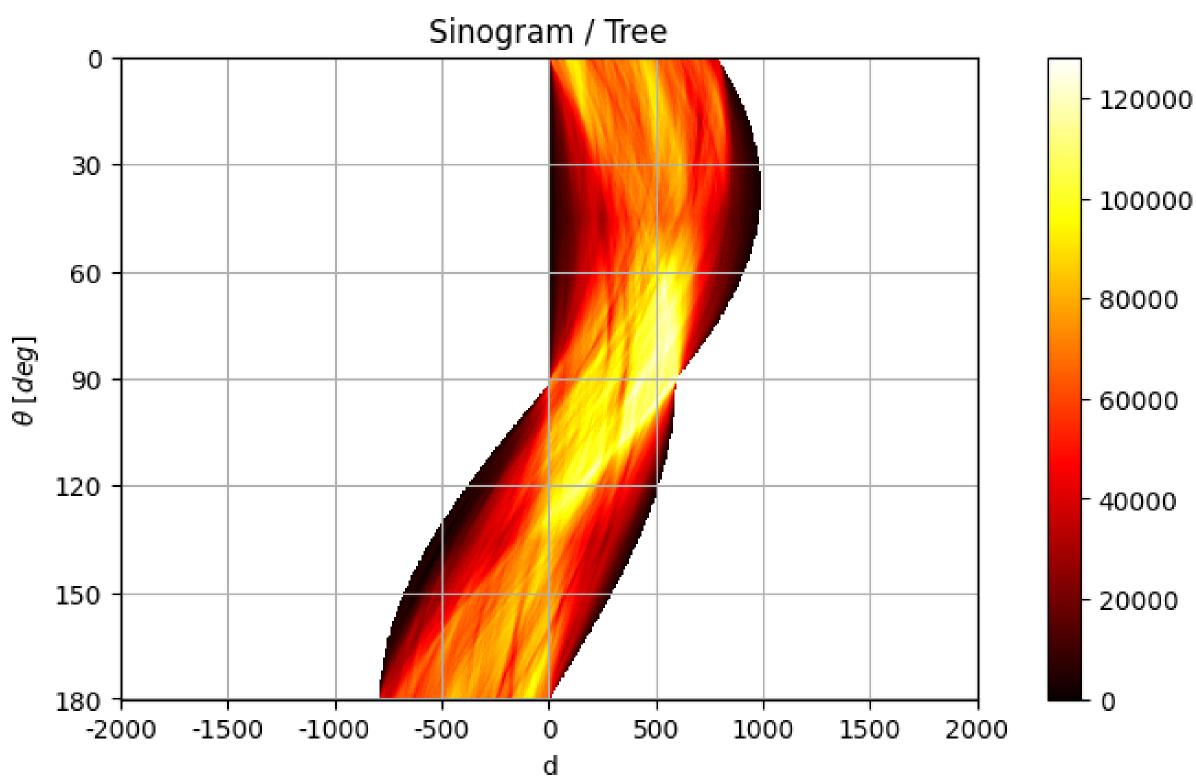
In [39]:
```python
imgFile2 = "images/tree_dublin.png"
img2 = cv2.imread(imgFile2, cv2.IMREAD_REDUCED_GRAYSCALE_2)
Nx2 = img2.shape[1]
Ny2 = img2.shape[0]
print(f"size of image: {img2.size} ; shape of image: {img2.shape}")

# Define boundaries of the rectangular region
x_l2 = 0  # left x
x_u2 = Nx2 - 1 # right x
y_l2 = 0 # bottom y
y_u2 = Ny2 - 1 # top y

d_min = -2000
d_max = 2000
Nd = 800
dVec = np.linspace(d_min, d_max, Nd)

Ntheta = 600
thetaVec_deg = np.linspace(0, 179, Ntheta)

# initalise matrix
sinogram2 = np.zeros((Ntheta, Nd), dtype=np.float64)

for nc, theta in enumerate(thetaVec_deg):
    # compute projection
    sinogram2[nc, :] = isec.projectionMultiLine(dVec, theta, img2, x_l2, x_u2, y_l2
```

```
size of image: 480000 ; shape of image: (600, 800)
```

In [40]:
```python
fig6 = plt.figure(6, figsize=[10, 10])
ax_f61 = fig6.add_subplot(2, 1, 1)

# plot of image
ax_f61.imshow(img2, cmap='Greys_r')
ax_f61.set_title("Tree")

# sinogram
ax_f62 = fig6.add_subplot(2, 1, 2)
a = ax_f62.imshow(sinogram2, cmap='hot' )
ax_f62.grid(True)
ax_f62.set_xlabel('d')
ax_f62.set_ylabel('$\\theta \ [deg]$ ')
ax_f62.set_title("Sinogram / Tree")

yticks = [0, 100, 200, 300, 400, 500, 599]
ytickLabels = ['0', '30', '60', '90', '120', '150', '180']
ax_f62.set_yticks(yticks, ytickLabels)

xticks = [0, 100, 200, 300, 400, 500, 600, 700, 800]
```

```
xtickLabels = ['-2000', '-1500', '-1000', '-500', '0', '500', '1000', '1500', '2000
ax_f62.set_xticks(xticks, xtickLabels)

fig6.colorbar(a, ax=ax_f62, location='right');
```





In [47]:

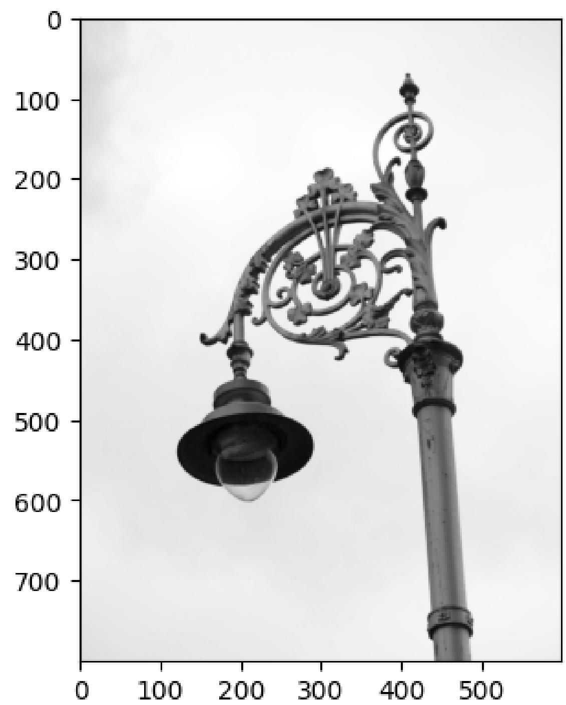
```
imgFile3 = "images/lamp_dublin_1.png"
img3 = cv2.imread(imgFile3, cv2.IMREAD_REDUCED_GRAYSCALE_2)
Nx3 = img3.shape[1]
Ny3 = img3.shape[0]
```

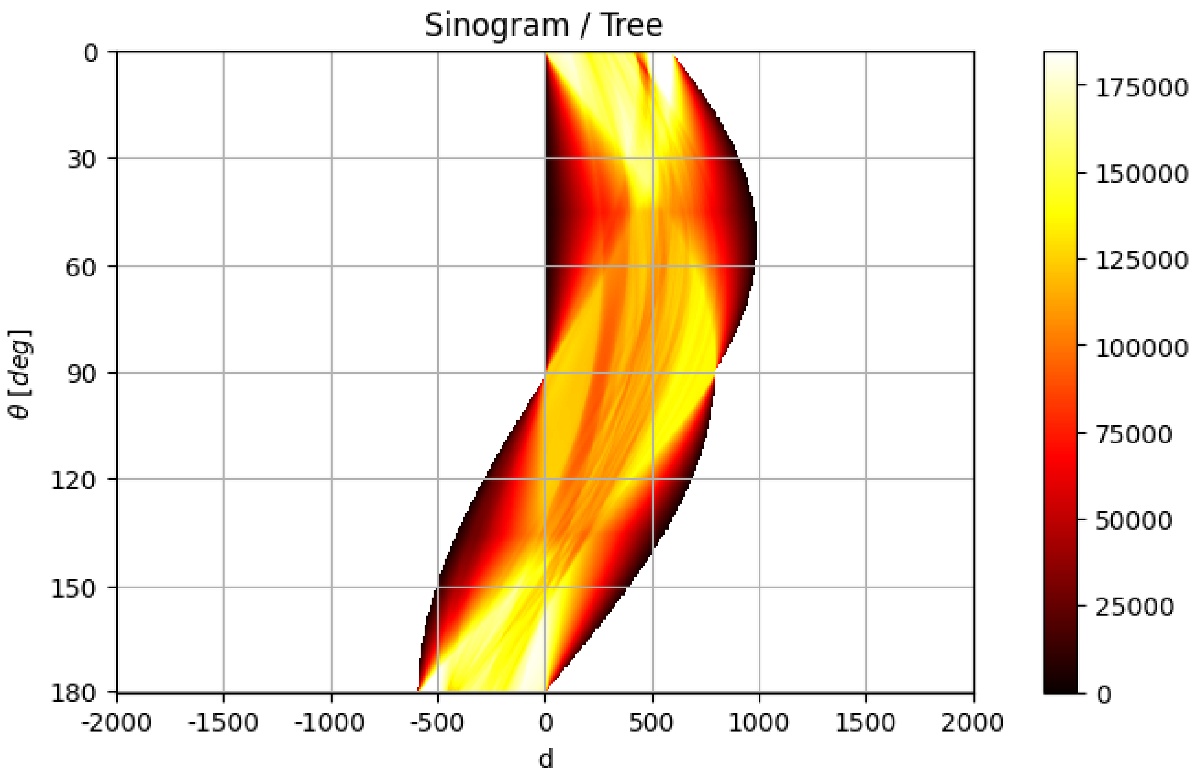
```
print(f"size of image: {img3.size} ; shape of image: {img3.shape}")

# Define boundaries of the rectangular region
x_l3 = 0  # left x
x_u3 = Nx3 - 1 # right x
y_l3 = 0 # bottom y
y_u3 = Ny3 - 1 # top y

d_min = -2000
d_max = 2000
Nd = 800
dVec = np.linspace(d_min, d_max, Nd)

Ntheta = 600
thetaVec_deg = np.linspace(0, 179, Ntheta)

# initalise matrix
sinogram3 = np.zeros((Ntheta, Nd), dtype=np.float64)

for nc, theta in enumerate(thetaVec_deg):
    # compute projection
    sinogram3[nc, :] = isec.projectionMultiLine(dVec, theta, img3, x_l3, x_u3, y_l3
```

```
size of image: 480000 ; shape of image: (800, 600)
```

In [48]:

```
fig7 = plt.figure(7, figsize=[10, 10])
ax_f71 = fig7.add_subplot(2, 1, 1)

# plot of image
ax_f71.imshow(img3, cmap='Greys_r')
ax_f71.set_title("Lamp")

# sinogram
ax_f72 = fig7.add_subplot(2, 1, 2)
a = ax_f72.imshow(sinogram3, cmap='hot' )

ax_f72.grid(True)
ax_f72.set_xlabel('d')
ax_f72.set_ylabel('$\\theta \ [deg]$ ')
ax_f72.set_title("Sinogram / Lamp")

yticks = [0, 100, 200, 300, 400, 500, 599]
ytickLabels = ['0', '30', '60', '90', '120', '150', '180']
ax_f72.set_yticks(yticks, ytickLabels)

xticks = [0, 100, 200, 300, 400, 500, 600, 700, 800]
xtickLabels = ['-2000', '-1500', '-1000', '-500', '0', '500', '1000', '1500', '2000
ax_f72.set_xticks(xticks, xtickLabels)

fig7.colorbar(a, ax=ax_f72, location='right');
```


0
100
200
300
400
500
600
700
0
100
200
300
400
500


Sinogram / Tree
θ [deg]
d
0
30
60
90
120
150
180
-2000
-1500
-1000
-500
0
500
1000
1500
2000
175000
150000
125000
100000
75000
50000
25000
0

In [ ]: